Every Student Counts

Middle School
Professional Development Guide
Year 2 - Day 3

Iowa Department of Education

Middle School Session –Facilitator Plan
Year 2 - Day 3

Content Goal:

NCTM Geometry Standards
Specify locations and describe spatial relationships using coordinate geometry and other representational systems
NCTM Measurement Standard
Apply appropriate techniques, tools, and formulas to determine measurements

Principle Focus: Teaching

Process Focus: Connections
Problem Solving
Overall Teaching Goal: Teaching and learning mathematics through problem solving

Activity	Description for Facilitator	Time (Min)	Teacher Masters (TM) & Materials
1. Welcome and opening	1. Welcome 2. Review Year 2 Outline 3. Review Daily Overview for Day 3 4. Review Day 3 agenda 5. React to teaching • PSSM (pp. 16 – 19) • How does Every Student Counts promote the Teaching Principle? 6. Review State ESC website • www.state.ia.us/educate/ecese/is/esc/index.hdml • user: ESCounts • Password: PBITSMDP • Review M Y2 D2 handout on paper and on the website • Review course requirements including homework, logs, meetings	45	**TM 1:** Year Two Outline **TM 2:** Daily Overview **TM 3:** Year 2 Day 3 Agenda **TM 4:** PBIT template **TM 5:** Team Log **TM 6:** Individual Report **TM 7:** Reflecting on Teaching Principle • Principles and Standards for School Mathematics (PSSM) • PSSM Quick Reference Guide

Activity	Description for Facilitator	Time (Min)	Teacher Masters (TM) & Materials
2. Debrief homework assignment	1. Homework review 2. Divide into two groups based on the assignment • Teach a middle school geometry PBIT (bring student work: high; middle; low) • Interview 3 or 4 middle school students about geometric understandings – 3. Find 2 other people that did the same assignment 4. Discuss lesson or interview in small groups 5. Share information in large group 6. Collect homework	30	**TM 8:** Van Hiele Levels **TM 9:** Homework Analysis
3. Meaningful Distributed Practice	1. Meaningful Distributed Practice (preview) 2. Design MDP activities	75	**TM 10:** MDP Activities **TM 11:** MDP Overheads **TM 12:** MDP Notes **TM 13:** MDP Components
4. Debrief Geometry Reading	1. Debrief Geometry Reading • What coordinate geometric concepts do middle school students generally understand when they leave elementary school? • What types of activities will help extend the middle school students/ understanding of coordinate geometry?	25	**TM 14:** Geometry Debriefing Questions

Activity	Description for Facilitator	Time (Min)	Teacher Masters (TM) & Materials
5. Problem-Based Instructional Task	1. Launch: Using a geoboard, review the last MDP activity and try to determine how we know if a figure is a square or not. 2. Explore: • Using graph paper, protractor and ruler, determine what makes a figure a square. o Explore minimal defensible definitions of a square. • Using a geoboard, determine characteristics of a square • Using a graphing calculator, determine characteristics of a square • Using Geometer's Sketchpad, determine characteristics of a square 3. Summarize: • What are the big ideas covered? • What tools did we use? When are they best used? • How can these tasks be modified or extended? • What are real-life applications? • How is this a PBIT? • How do the Van Hiele levels impact the use of these activities?	135	**TM 15:** Problem-Based Instructional Task **TM 16:** Properties of Quadrilaterals **TM 17:** Setting up Geoboard **TM 18:** GSP Assignment **TM 19:** GSP Extension **TM 20:** GSP Extension Answers **TM 21:** Geoboard with rhombus **TM 22:** PBIT Components **TM 23:** Summarize Overheads • Ruler • Protractor • http://education.ti.com/guidebooks/apps/73geoboard/ti73geoboard.pdf • Overhead graphing calculator • Geoboard • Geobands • Graph paper • Geometer Sketchpad • Graph paper available at two web sites o http://www.mathematicshelpcentral.com o http://www.mathsphere.co.uk/Resources/MathSphereFreeGraphPaper.shtml

Activity	Description for Facilitator	Time (Min)	Teacher Masters (TM) & Materials
6. Closure	1. Review day's agenda 2. Review assignment for April 11/12	15	**TM 1**: Overview **TM 24**: Assignment **TM 25:** Evaluation

Facilitator's Tool for Planning the Session

What is the background reading?

1. *NCTM Principles and Standards* – Teaching (pp. 16-19)
2. *NCTM Principles and Standards* – Geometry (pp. 232-239)

What equipment and materials should **participants** bring?

- Computer
- Graphing Calculator
- *Navigating through Algebra in Grades 6 – 8* book and CD
- Principles and Standards for School Mathematics (PSSM)
- Ruler

What Teaching Masters need to be copied?

Handouts:

TM 1: Year Two Outline
TM 2: Daily Overview
TM 3: Year 2 Day 3 Agenda
TM 4: PBIT template
TM 5: Team Log
TM 6: Individual Report
TM 8: Van Hiele Levels
TM 9: Homework Analysis
TM 10: MDP Activities
TM 11: MDP Overheads
TM 12: MDP Notes
TM 13: MDP Components
TM 14: Geometry Debriefing Questions
TM 15: Problem-Based Instructional Task
TM 16: Properties of Quadrilaterals
TM 17: Setting up Geoboard
TM 18: GSP Assignment
TM 19: GSP Extension
TM 20: GSP Extension Answers

TM 21: Geoboard with rhombus
TM 22: PBIT Components
TM 23: Summarize Overheads
TM 24: Assignment
TM 25: Evaluation

What Teaching Masters need to be copied for presenters?

TM 1: Year Two Outline
TM 2: Daily Overview
TM 3: Year 2 Day 3 Agenda
TM 4: PBIT template
TM 5: Team Log
TM 6: Individual Report
TM 7: Reflecting on Teaching Principle
TM 8: Van Hiele Levels
TM 9: Homework Analysis
TM 10: MDP Activities
TM 11: MDP Overheads
TM 12: MDP Notes
TM 13: MDP Components
TM 14: Geometry Debriefing Questions
TM 15: Problem-Based Instructional Task
TM 16: Properties of Quadrilaterals
TM 17: Setting up Geoboard
TM 18: GSP Assignment
TM 19: GSP Extension
TM 20: GSP Extension Answers
TM 21: Geoboard with rhombus
TM 22: PBIT Components
TM 23: Summarize Overheads
TM 24: Assignment
TM 25: Evaluation

Teaching supplies/materials/technologies

- Isometric and grid graph paper available at two web site
  - http://www.mathematicshelpcentral.com
  - http://www.mathsphere.co.uk/Resources/MathSphereFreeGraphPaper.shtml
- Link to State website
  - www.state.ia.us/educate/ecese/is/esc/index.hdml

- Geobands

• Geoboard
• Geometer Sketchpad
• Graph paper
• Overhead graphing calculator
• Principles and Standards for School Mathematics (PSSM)
• Protractor
• PSSM Quick Reference Guide
• Ruler

Activity 1: Welcome and Opening

Time: 45 minutes

Overview and Rationale:
This activity connects the day with the goals for the year. It will provide an opportunity to relate daily activities to the year-long goals and activities.

Conducting the Activity:

1. Year 2 Outline Chart TM 1
   - Remind participants of the big picture for the year
   - Point out where we've been and where we're going
   - Emphasize the NCTM Content Standards, Principles and Process Standards of the day
2. Go through Year 2 Day 3 Agenda TM 3 handout while using the Day 3 Overview Chart TM 2
   - Briefly go through agenda
   - Remind participants of the main themes of Every Student Counts
   - Point out how those themes will be applied to the goals and focus areas
   - Use the Quick Reference Guide to locate the NCTM Standards being highlighted
3. Discuss Principle Focus
   - Participants discuss TM 4 – Reflecting on Teaching
   - Share a few ideas from each team with the whole group
4. View State Website
   - www.state.ia.us/educate/ecese/is/esc/index.hdml
   - Review passwords
     - user: ESCounts
     - Password: PBITSMDP
   - Review M Y2 D2 handout that is also on the website
5. Review PBIT, Team Log and course requirements

Materials

TM 1: Year Two Outline
TM 2: Daily Overview
TM 3: Year 2 Day 3 Agenda
TM 4: PBIT template
TM 5: Team Log
TM 6: ESC Individual Report
TM 7: Reflecting on Teaching Principle

- Principles and Standards for School Mathematics (PSSM)
- PSSM Quick Reference Guide

TM 1

Year 2 Outline
2005-2006

	Day 1 October 4/5	Day 2 November 8/9	Day 3 January 31/February 1	Day 4 April 11/12
NCTM Content Standard	Geometry	Geometry	Geometry	Geometry
	Analyze characteristics and properties of 2 and 3-dimensional shapes and develop mathematical arguments about geometric relationships	Use visualization, spatial reasoning, and geometric modeling to solve problems	Specify locations and describe spatial relationships using coordinate geometry and other representational systems	Apply transformations and use symmetry to analyze mathematical situations
NCTM Content Standard 2	Geometry	Measurement	Measurement	
	Use visualization, spatial reasoning, and geometric modeling to solve problems	Apply appropriate techniques, tools, and formulas to determine measurements	Apply appropriate techniques, tools, and formulas to determine measurements	
NCTM Content Standard 3	Measurement			
	Understand measurable attributes of objects and the units, systems, and processes of measurement			
Mathematical Activities	Analyzing characteristics and properties of polygons	Use visualization, spatial reasoning, and geometric modeling Maximizing and minimizing area, perimeter and volume	Use coordinate geometry to represent and examine the properties of geometric shape	Apply transformations and use symmetry to analyze mathematical situation
NCTM Principle	Equity	Technology	Teaching	Learning

	Day 1 **October 4/5**	**Day 2** **November 8/9**	**Day 3** **January 31/February 1**	**Day 4** **April 11/12**
NCTM Process Standard	Communication Reasoning and Proof Problem Solving	Communication Representation Problem Solving	Connections Problem Solving Reasoning and Proof	Problem Solving Connections
Assessment	Teacher Observation Checklist to provide Feedback to the Students Questioning	Rubric Collecting Samples of Participants' Work Analyzing Student Work to Provide Feedback to the Students	Formative use of summative assessment Examining ITBS and classroom summative assessment	Peer Assessment
Technology/ Manipulative Tools	Sketchpad Navigation CD Computer (participants bring) Geostrips	Graphing Calculator Navigation CD – Applet Cubes Web Page: http://illuminations.nctm.org/index.aspx.	Graphing Calculator Geoboards Geometry Sketchpad http://education.ti.com/guidebooks/apps/73geoboard/ti73geoboard.pdf	Applet Sketchpad Geoboard Georeflector Computer (participants bring)

TM 2

Teach for Understanding and Focus on Meaning

Problem-Based Instructional Tasks Teaching through Problem Solving	Meaningful Distributed Practice of Concepts, Skills, & Problem Solving

Today's Goals . . .

Content Goal: Geometry and Measurement
Principle Goal: Teaching
Process Goals: Connections
Problem Solving
Reasoning and Proof

Today's Objectives . . .

- *Specify locations and describe spatial relationships using coordinate geometry and other representational systems*
- *Apply appropriate techniques, tools, and formulas to determine measurements*

TM 3

Year 2 Day 3 Agenda

- Welcome and opening
- Debrief Homework Assignment
- Meaningful Distributive Practice
- Debrief Geometry Reading
- Problem-Based Instructional Task
- Closure

Assignments for April 11/12:

1. *NCTM Navigating Through Geometry in Grades 6 - 8* – Transformation (pp. 43-44)
   - What are the important ideas in the transformations and symmetry section of the geometry standard?
   - What are some things that might cause problems to students in this area?

2. *NCTM Principles and Standards* – Learning (pp. 20-21) Cognitive research tells us that students learn best when they have an opportunity to build on prior experience and knowledge, develop conceptual understanding during relevant problem solving experiences and use metacognitive reflection.
   - How can these principles enhance student engagement?

3. Slavit, David. "Above and beyond AAA: The Similarity and Congruence of Polygons." on CD from *Navigating through Geometry in Grades 6 – 8* – Article is under section called "More Readings."
   - What impact does this article suggest for curriculum being taught?
   - What impact does this article suggest for instruction?

4. *Driscoll, Mark.* "The Sound of Problem Solving*."* *Teaching Mathematics through Problem Solving.* (pp. 161 – 175).
    - What can you gain from listening to students when they are working in groups?

5. Brown, Scott. "You Made It through the Test; What about the Aftermath*?"* *Mathematics Teaching in the Middle School*. (pp. 68 – 73).
    - How does this promote formative use of summative assessment?

6. Do the following activity: Observe a teacher teaching a PBIT
    - List any questions that were asked in class
    - Who asked the questions?
    - Describe the ways the lesson was like a PBIT and the ways it was not
    - How did the teacher assess the students?

Resources to bring on April 11/12:

An email will be sent listing resources to bring in April.

TM 4

PROBLEM-BASED INSTRUCTIONAL TASK LESSON PLAN

OBJECTIVE/BENCHMARK:

TITLE:

GRADE LEVEL/COURSE: Middle School

PRE-REQUISITE KNOWLEDGE:

NCTM STANDARD(S): (Shaded)

NCTM Content Standards →	Number & Operations	Algebra	Geometry	Measurement	Data Analysis & Probability
NCTM Process Standards →	Problem Solving	Reasoning & Proof	Communication	Connections	Representation

MATERIALS NEEDED:

Audio-visual:
Manipulatives/Materials:
Literature:
Technology/Software:
Other:

MAIN LESSON DEVELOPMENT:

- **Launch**
- **Explore**
- **Summarize**

MODIFICATIONS/EXTENSIONS:

- **Modifications**
- **Extensions**

CHECKING FOR UNDERSTANDING (FORMATIVE ASSESSMENT)

- **What will you assess?**
- **How will you assess it?**

---------- (REFLECTION AFTER TEACHING THE LESSON) ----------

- **How did the students perform?**
- **How will you use this information to guide future instructional decisions?**

TM 5

Every Student Counts
AEA/Urban 8 Math Team Meeting Log

Name of Team	**Date of Meeting**
Team Members Attending % _____	**Time Started** **Time Ended**

1. Process homework assignments.

2. Update regarding collaboration with classroom teaching partners
   Each member should share observation/lesson plan for their teaching practice with the group.
   Group members should offer additional support, suggestions, etc. to the person sharing.

Participant	Lesson topic	Grade level	Date

3. Other agenda items specific to your AEA/Urban 8 Math Team.

4. Next meeting – Decide on the following:

   - Date: _____
   - Place: _____
   - Facilitator: _____
   - Recorder: _____
   - Agenda: _____

TM 6

Every Student Counts
Individual Report
2005-2006

Name:_____

Your position:_____

Name of your organization:_____

What level of Every Student Counts?:_____

Dates you attended our PD days:_____

_____

Dates you attended your team meetings: (**provide copies of those agendas**)

_____

_____

Name, grade level taught, and school name of your partner teacher:

_____

_____

Dates you were in your partner teacher's classroom and what you were doing (ex. Teaching a PBIT, observing, working with individual students)

_____

_____

TM 7

Reflecting on Teaching

(pp. 16 - 19 PSSM)

How does Every Student Counts promote the Teaching Principle?

Activity 2: Homework Analysis

Time: 30 minutes

Overview and Rationale

This activity provides an opportunity to share

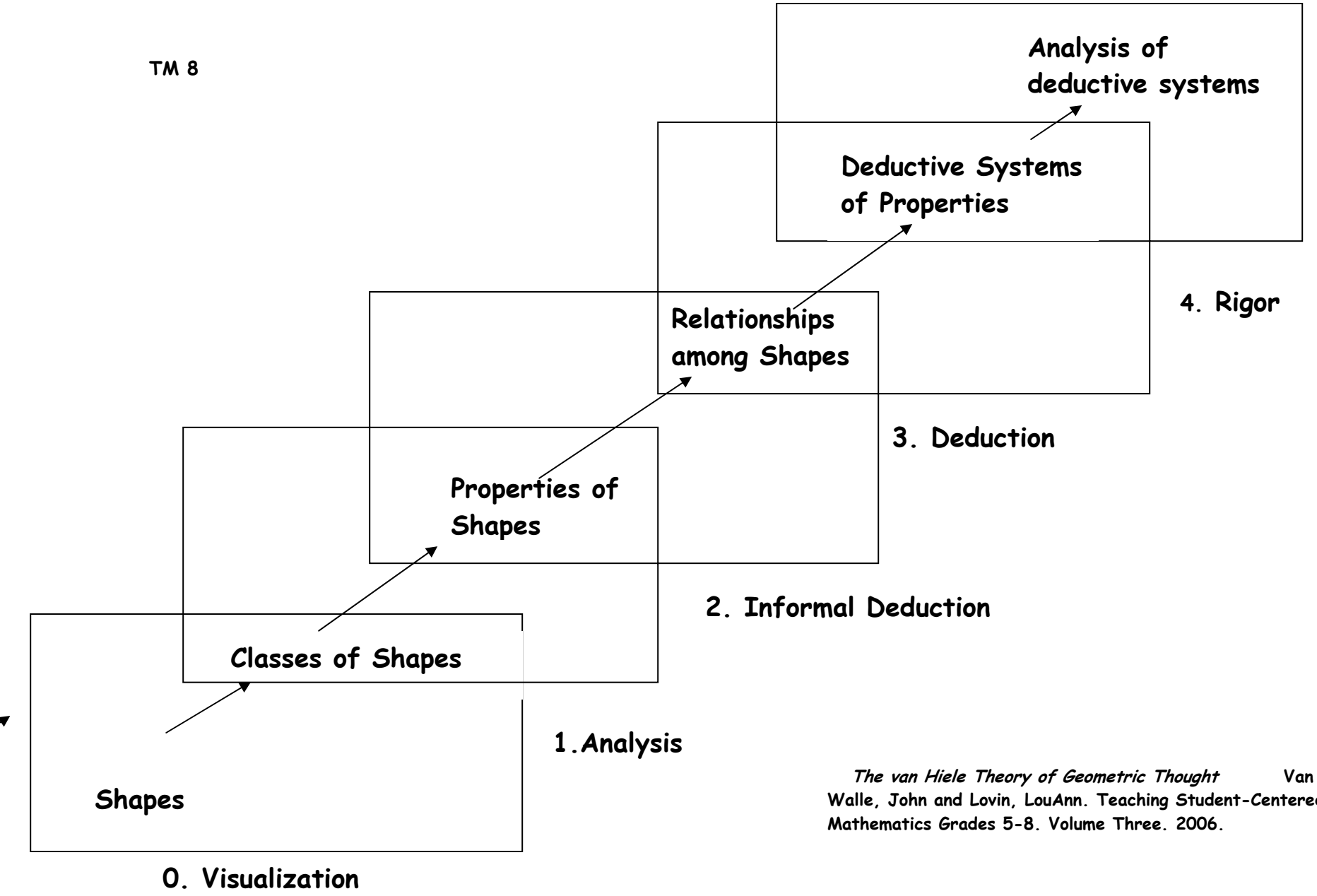
Conducting the Activity

1. Find 2 other people that did the same assignment
   - Teach a middle school geometry PBIT (bring student work: high; middle; low)
   - Interview 3 or 4 middle school students about geometric understandings – Bring list of student comments and/or work activities that illustrate students working at different van Hiele levels
2. Discuss lesson or interview in small groups
3. Share information in large group
4. Collect homework

Materials

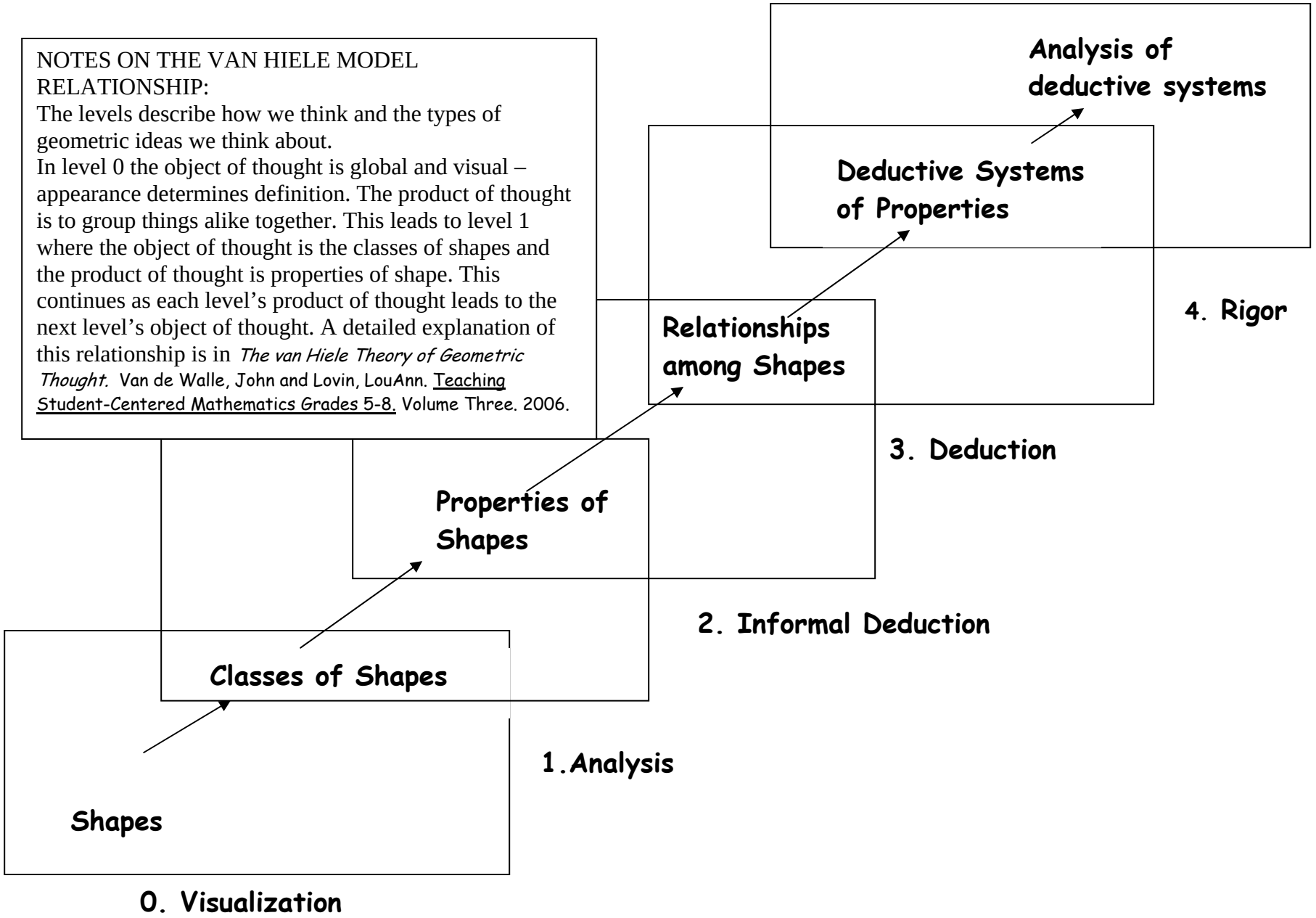
TM 8: Van Hiele Levels
TM 9: Homework Analysis

TM 8



The van Hiele Theory of Geometric Thought Van de Walle, John and Lovin, LouAnn. Teaching Student-Centered Mathematics Grades 5-8. Volume Three. 2006.

NOTES ON THE VAN HIELE MODEL RELATIONSHIP:
The levels describe how we think and the types of geometric ideas we think about.
In level 0 the object of thought is global and visual – appearance determines definition. The product of thought is to group things alike together. This leads to level 1 where the object of thought is the classes of shapes and the product of thought is properties of shape. This continues as each level's product of thought leads to the next level's object of thought. A detailed explanation of this relationship is in *The van Hiele Theory of Geometric Thought.* Van de Walle, John and Lovin, LouAnn. Teaching Student-Centered Mathematics Grades 5-8. Volume Three. 2006.



TM 9

Homework Analysis

1. **Discuss lesson or interview**
2. **What did you learn about students' thinking?**
3. **What connections were you able to make to the Van Hiele levels?**

Level 0: Visual or Concrete

Shapes and what they look like

Level 1: Analysis

Properties of shapes

Level 2: Informal Deduction

Relationships among properties

Level 3: Deduction

Deductive axiomatic systems

Level 4: Rigor

Comparisons and contrasts among axiomatic systems

Activity 3: Meaningful Distributed Practice (MDP)

Time: 75 minutes

Overview and Rationale

This activity provides practice with Meaningful Distributed Practice. It connects MDP with geometry goals and assessment.

Conducting the Activity

1. Complete the MDP activities
2. Review reason why specific activities were chosen
3. Have participants write MDP

Materials

TM 10: MDP Activities
TM 11: MDP Overheads
TM 12: MDP Notes
TM 13: MDP Components

TM 10

Meaningful Distributed Practice: Grade Level/Class

Big Idea(s) Coordinate Geometry

Day One	Day Two	Day Three
Practice Activity 1 Graph the points (4,5), (7,5), (7,2), (4,2).	Practice Activity 2 Given points on geoboard: (0,4) (3,4) (3,1) Find the 4th point that would make a square.	Practice Activity 3 Given points as ordered pairs: (1,1) (5,1) (5,5) Find the 4th point that would make a square.
Questions: What shape is this? How do you know?	Questions: What is the area? How did you know where the 4th point would be? How did you find the area of the square? Could you do this without the geoboard? What do you notice about the x coordinates? What do you notice about the y coordinates?	Questions: What is the area? How did you know where the 4th point would be? How can you be sure this is a square? How did you find the area of the square? How are the x coordinates related? How are they coordinates related?

Day Four	Day Five	Day Six
Practice Activity 4 Put marker on overhead to represent two opposite vertices of a square. (7,0), (4,3) Give coordinates of the two remaining vertices.	Practice Activity 5 Give ordered pairs (0,5), (3,8) Give coordinates of the two remaining vertices.	Practice Activity 6 Give ordered pairs (2,3), (4,5), (6,3). Find the 4th vertex needed to make a square.
Questions: What is the area of this square? What is the perimeter of this square? How did you determine the vertices? Are there any other opposite vertices that make a square? How do you determine the area? How do you determine the perimeter?	Questions: What is the area of this square? What is the perimeter of this square? How did you determine the vertices? Are there any other opposite vertices that make a square? How do you determine the area? How do you determine the perimeter?	Questions: How did you know the vertex? How do you know this is a square?

TM 11

Meaningful Distributed Practice - Overhead

Graph the points (4, 5), (7, 5), (7, 2), (4, 2).

Given the points (0, 4), (3, 4), (3, 1) on the geoboard, find the fourth point that would make a square. What is the area of the square?

Given the points (1, 1), (5, 1), (5, 5), find the fourth point that would make a square. What is the area of the square?

Given ordered pairs (7,0) and (4,3), find possible coordinates of the two remaining vertices. What is the area of this square? What is the perimeter of this square?

Given ordered pairs (0,5) and (3,8), find possible coordinates of the two remaining vertices. What is the area of this square? What is the perimeter of this square?

Given the ordered pairs (2, 3), (4, 5), (6, 3), find the fourth vertex needed to make a square.

TM 12

NOTES
MEANINGFUL DISTRIBUTED PRACTICE

Graph the points (4, 5), (7, 5), (7, 2), (4, 2).

- What shape is this? *Square*
- How do you know? *Four congruent sides, four right angles*

Process: *Why would I ask this question? What skills am I trying to assess? I am looking to see if students can plot points, and identify a square and characteristics of a square. This can be used to assess Van Hiele level.*

Given the points (0, 4), (3, 4), (3, 1) on the geoboard, find the fourth point that would make a square. *(0, 1)* What is the area of the square? *9 units*

- How did you know where the fourth point would be? *Points line up, coordinates can be used, make it look like a square.*
- How did you find the area of the square? *Count the squares, multiply using the formula.*
- Could you do this without a geoboard? *Use the formula.*
- What do you notice about the x-coordinates? *Points that are above each other have the same x-coordinate.*
- What do you notice about the y coordinates? *Points that are across from each other have the same y-coordinate.*

Process: *Why would I ask this question? What skills am I trying to assess? Visualize a square, look for relationships between the coordinates, and conceptually review area. and connect the formula.*

Given the points (1, 1), (5, 1), (5, 5), find the fourth point that would make a square. What is the area of the square?

- How did you know where the fourth point would be?
- How can be sure this is a square?
- How did you find the area of the square?

- How are the x-coordinates related?
- How are the y-coordinates related?

Don't actually do this problem. This is like the last problem, but without the geoboard. Why would I ask this problem without the geoboard? We want to encourage the students to visualize the square, move students towards the area formula, and reinforce the relationships between the x and y coordinates.

(Put marker on overhead to represent two opposite vertices of a square: (7, 0) and (4, 3)). Give the coordinates of the two remaining vertices. *(7,0) and (4, 3)* What is the area of this square? *9* What is the perimeter of this square? *12*

- How did you determine the vertices? *Followed lines up and across to find the points or used the x and y-coordinates from the given points.*
- How did you determine the area? *Count squares or formula*
- How did you determine the perimeter? *Count squares or formula*
- Are there any other opposite vertices that make a square? *(10, 3) and (7, 6)*

Process: *Why would we move to a problem with only two points? We want to push the students to visualize the square and push the understanding of the relationship of the coordinates. This will also encourage students to further develop their intuitive understanding of the characteristics of a square. Why would we ask students to find another possible square? This encourages students to look beyond the obvious solution and to experience squares with different orientations.*

(Give the ordered pairs (0, 5) and (3, 8)).
Give the coordinates of the two remaining vertices. What is the area of this square? What is the perimeter of this square?

- How did you determine the vertices?
- How did you determine the area?
- How did you determine the perimeter?

- Are there any other opposite vertices that make a square?

Don't actually do this problem. This is like the last problem, but without the geoboard. Why would I ask this problem without the geoboard? We want to encourage the students to visualize the square, move students towards the formulas, and experience squares with different orientations.

Given the ordered pairs (2, 3), (4, 5), (6, 3), find the fourth vertex needed to make a square. *(4, 1)*

- How did you know where the fourth point would be? *Use the diagonals to find the missing point, make the sides the same length, and make the angles look right.*

Participants practice writing Meaningful Distributed Practice.

Now that we've gone through this series of distributed practice, you're going to get a chance to write distributed practice problems for perimeter and area.

Last time we did distributed practice on perimeter and area by having students draw representations of rectangles and consider different rectangles with the same perimeter. There are many ways to model perimeter and area and one way we are using today is the geoboard.

(Put the geoboard on the overhead showing one tilted square.) What is the area of this figure? *2*

At your tables, design a problem that could follow this problem in a distributed practice series.

(Have the participants share several examples. Choose one to use as the second problem in the series, preferably one that moves towards area of triangles. Have the participants use this problem to plan for the third problem in the series. Continue this for a five day series. Collect what the participants have done.)

TM 13

Meaningful Distributed Practice of Concepts, Skills and Problem-Solving

- Targets an identified need based on multiple data sources
- Helps students develop a deep understanding of a *Big Idea*
- Helps students develop flexibility and fluency with skills and concepts
- Builds on and extends understanding
- Uses problems and activities that help students learn to use multiple representations and learn to use multiple reasoning strategies
- Uses problems from a variety of contexts so students learn to make connections.

Revised September 2005

Activity 4: Debrief Geometry Reading

Time: 25 minutes

Overview and Rationale

This activity highlights the concepts middle school students understand and illustrates different types of activities which help students develop geometric concepts.

Conducting the Activity

1. Participants at half the tables discuss the coordinate geometry concepts middle school students generally understand when they leave elementary school.
2. Participants at half the tables discuss the types of activities that will help extend the middle school students' understanding of coordinate geometry.
3. Each table shares their thoughts

Materials

TM 14: Geometry Debriefing Questions

TM 14

Reflecting on Geometry

What coordinate geometry concepts do middle school students generally understand when they leave elementary school?

What types of activities will help extend the middle school students' understanding of coordinate geometry?

Activity 5: Problem-Based Instructional Task: Square Pegs

Time: 135 minutes including break

Overview and Rationale

This activity will develop reasoning about squares.

Conducting the Activity

LAUNCH

How do we know this is a square?

(3,2), (7,2), (7,6), (3,6)

1. How do you know this is a square?
   - How do you know it's a right angle?
   - How do you know the sides are congruent?
2. What are some other characteristics of squares?
   - What about the diagonals?
     - Diagonals are perpendicular
     - Diagonals are congruent
     - Diagonals bisect each other
   - Opposites sides parallel

EXPLORE

How do we know this is a square?

(2,3) (4,5) (6,3) (4,1)

1. Give graph paper. protractor, ruler
   a. Use physical measurement tools
   b. Is this enough for this to be a square?
   c. Is this enough?
2. Go to geoboard
3. Go to graphing calculator
   http://education.ti.com/guidebooks/apps/73geoboard/ti73geoboard.pdf
4. Go to sketchpad (30)
   a. Give settings for sketchpad
   b. Measure angles
   c. Measure lengths
   d. Measure slopes

SUMMARIZE

1. What are the big ideas covered?
2. What tools did we use? Which tools worked best for which situations?
3. How can these tasks be modified or extended?
4. Can we find real-life applications?
5. What makes this a PBIT?
6. How do the Van Hiele levels impact how these activities are used?

Activity 5 (Continued)**: Problem-Based Instructional Task: Square Pegs**

Materials

TM 15: Problem-Based Instructional Task
TM 16: Properties of Quadrilaterals
TM 17: Setting up Geoboard
TM 18: GSP Assignment
TM 19: GSP Extension
TM 20: GSP Extension Answers
TM 21: Geoboard with rhombus
TM 22: PBIT Components
TM 23: Summarize Overhead and Results

- Ruler
- Protractor
- Overhead graphing calculator
- Geoboard
- http://education.ti.com/guidebooks/apps/73geoboard/ti73geoboard.pdf
- Geobands
- Graph paper
- Geometer Sketchpad
- Graph paper available at two web sites
  - http://www.mathematicshelpcentral.com
  - http://www.mathsphere.co.uk/Resources/MathSphereFreeGraphPaper.shtml

TM 15

PROBLEM-BASED INSTRUCTIONAL TASK

OBJECTIVE/BENCHMARK:

Geometry

- Analyze characteristics and properties of two-dimensional geometric shapes and develop mathematical arguments about geometric relationships
  - Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties

Measurement

- Apply appropriate techniques, tools, and formulas to determine measurements
  - Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision

TITLE: Square Pegs

GRADE LEVEL/COURSE: Middle School

PRE-REQUISITE KNOWLEDGE:

Knowledge of diagonal, right angles, congruent, square

NCTM STANDARD(S): (Shaded)

NCTM Content Standards →	Number & Operations	Algebra	Geometry	Measurement	Data Analysis & Probability
NCTM Process Standards →	Problem Solving	Reasoning & Proof	Communication	Connections	Representation

MATERIALS NEEDED:

Audio-visual:
Manipulatives/Materials:
geobands
geoboard
graph paper
overhead geoboard
protractor
Literature:
Technology/Software:
TI73 calculators
Geometer Sketchpad
http://education.ti.com/guidebooks/apps/73geoboard/ti73geoboard.pdf
Other:

MAIN LESSON DEVELOPMENT:

Launch:

1. Use the geoboard and model (3,2), (7,2), (7,6), (3,6)
   - How do we know this is a square?
     - *How do you know the angle is a right angle?*
     - *How do you know the sides are congruent?*
   - How can you convince us that it is a square?
   - Can you get by with less?
   - Can you come up with a minimal defensive definition?
     - *Minimal - If a single property is removed from the definition, it no longer defines the object.*
     - *Defensive – If a counterexample can be found, it is not defining.*
   - What about diagonals make this a square?
     - *If diagonals are perpendicular and congruent, figure is a kite.*
     - *If diagonal bisect one another, figure is a rhombus.*
     - *If diagonals bisect one another and are congruent, figure is a rectangle.*
     - *If diagonals are perpendicular, congruent, and bisect one another, figure is a square*

Explore:

1. Using big graph paper, a protractor, and a ruler, draw this figure.
   (4,10) (11,17) (18,10) (11,3)
   - There was little question that the first figure was a square, how do you know this figure is a square?
   - Have students share their reasons and ask others if this is enough for this to be a square? If students have counterexamples, have them bring them to the overhead to share.
2. Go to graphing calculator
   - http://education.ti.com/guidebooks/apps/73geoboard/ti73geoboard.pdf
     - Pp 6 through 13 and pp. 31 – 32
   - How would you use this tool to talk about characteristics of a square?
   - How would you use this tool to promote learning and thinking?
3. Go to Geometer's Sketchpad
   - Give settings for sketchpad
     - Measure angles
     - Measure lengths
     - Measure slopes
   - How would you use this tool to talk about characteristics of a square?
   - How would you use this tool to promote learning and thinking?

Summarize:

1. What are the big ideas covered?
2. What tools did we use? Which tools worked best for which situations?
3. How can these tasks be modified or extended?

4. Can we find real-life applications?
5. What makes this a PBIT?
6. How do the Van Hiele levels impact how these activities are used?

MODIFICATIONS/EXTENSIONS:

"Properties of Quadrilaterals" *NCTM Addenda Series 5-8*

CHECKING FOR UNDERSTANDING (FORMATIVE ASSESSMENT)

What will you assess?

- Properties of squares
- Measuring sides and angles

How will you assess it?

- During student problem solving time, teacher observation will determine whether students can correctly measure sides and angles

- Students will write what they see as the essential components of a definition of a square

---------- (REFLECTION AFTER TEACHING THE LESSON) ----------

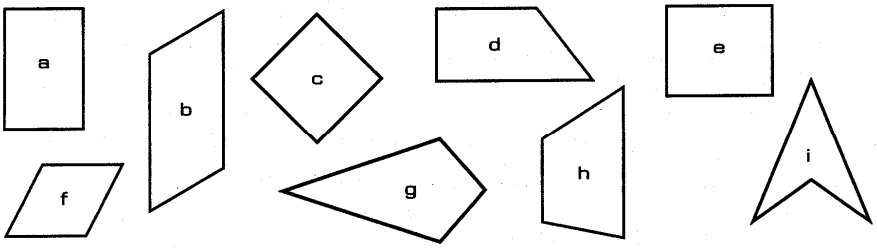
- **How did the students perform?**

- **How will you use this information to guide future instructional decisions?**

TM 16

ACTIVITY 13C
PROPERTIES OF QUADRILATERALS

Select the figures that satisfy each of the sets of properties below and state their names.



Has 4 sides. Opposite sides are congruent. Opposite sides are parallel. Opposite angles are congruent. Figures:_____ Name:_____	Has 4 sides. Only two sides are parallel. Figures:_____ Name:_____
Has 4 sides. All sides are congruent. Opposite sides are parallel. All angles are right angles. Figures:_____ Name:_____	Has 4 sides. Has two pairs of congruent adjacent sides. Figures:_____ Name:_____
Has 4 sides. Opposite sides are congruent. Opposite sides are parallel. All angles are right angles. Figures:_____ Name:_____	Has 4 sides. All sides are congruent. Opposite sides are parallel. Opposite angles are congruent. Figures:_____ Name:_____

Properties	Parallelogram	Rectangle	Rhombus	Square	Trapezoid	Kite
Diagonals bisect each other.						
Diagonals are congruent.						
Diagonals are perpendicular.						
Diagonals bisect vertex angles.						
One diagonal forms two ≅ triangles.						
Diagonals form four ≅ triangles.						

TM 17

Setting Up the Geoboard on Geometer's Sketchpad

Edit → Preferences

Under units tab, be sure that the distance is set to centimeter. Measure to the nearest tenth. Click OK.

Graph → Define coordinate system

Select the coordinate system by clicking on a grid intersection. For example, click on the point (1, 1). The grid should be purple indicating that it is selected.

Display → Line Width → Dotted

This changes the coordinate lines to dots.

Graph → Snap Points

This command causes independent points to snap to nearby locations when you drag them.

Move the origin

The geoboards have coordinates that correspond to the first quadrant of a graph. To make the GSP geoboard like the manipulative, click and hold the origin point. Move it so the origin is on the lower, left side of the screen.

Display → Hide

This will hide various items on your screen, depending on what is selected. To hide the origin and the unit point, select the two points then under display choose hide points. To hide the axes, select them and choose hide axes.

TM 18

GSP Assignment:

Use the point tool to place four points on your Geoboard. Use the select tool to select all four points.

Construct → Segments

This should construct segments between the four points, making a quadrilateral. Arrange the points so you have a square.

Measure

If you select a side, you should be able to measure length and slope. If you select three points, you should be able to measure angles. (If you have trouble measuring, double check what you have selected. You may have the previous measurement selected or an additional point, etc.)

Identify as many characteristics as you can to verify that the shape you have created is a square.

TM 19

GSP Extension:

Create a square on your geoboard. Construct the diagonals and identify as many characteristics of the diagonals of a square as you can. Change the square to a rhombus (you will probably have to un-select "snap points" under graph). Identify as many characteristics of the diagonals of a rhombus as you can. Continue with a rectangle and a parallelogram. Use the graphic organizer to help you. Complete the statements when you are finished.

Square	Rhombus	Rectangle	Parallelogram

If the diagonals of a quadrilateral _____, then the quadrilateral must be a parallelogram.

If the diagonals of a quadrilateral _____, then the quadrilateral must be a rectangle.

If the diagonals of a quadrilateral _____, then the quadrilateral must be a rhombus.

If the diagonals of a quadrilateral _____, then the quadrilateral must be a square.

Bonus: create a Venn diagram showing the relationships between the quadrilaterals based on the characteristics of their diagonals.

TM 20

GSP Extension Answers:

Create a square on your geoboard. Construct the diagonals and identify as many characteristics of the diagonals of a square as you can. Change the square to a rhombus (you will probably have to un-select "snap points" under graph). Identify as many characteristics of the diagonals of a rhombus as you can. Continue with a rectangle and a parallelogram. Use the graphic organizer to help you. Complete the statements when you are finished.

Square	Rhombus	Rectangle	Parallelogram
Congruent Perpendicular Bisect each other	Perpendicular Bisect each other	Congruent Bisect each other	Bisect each other

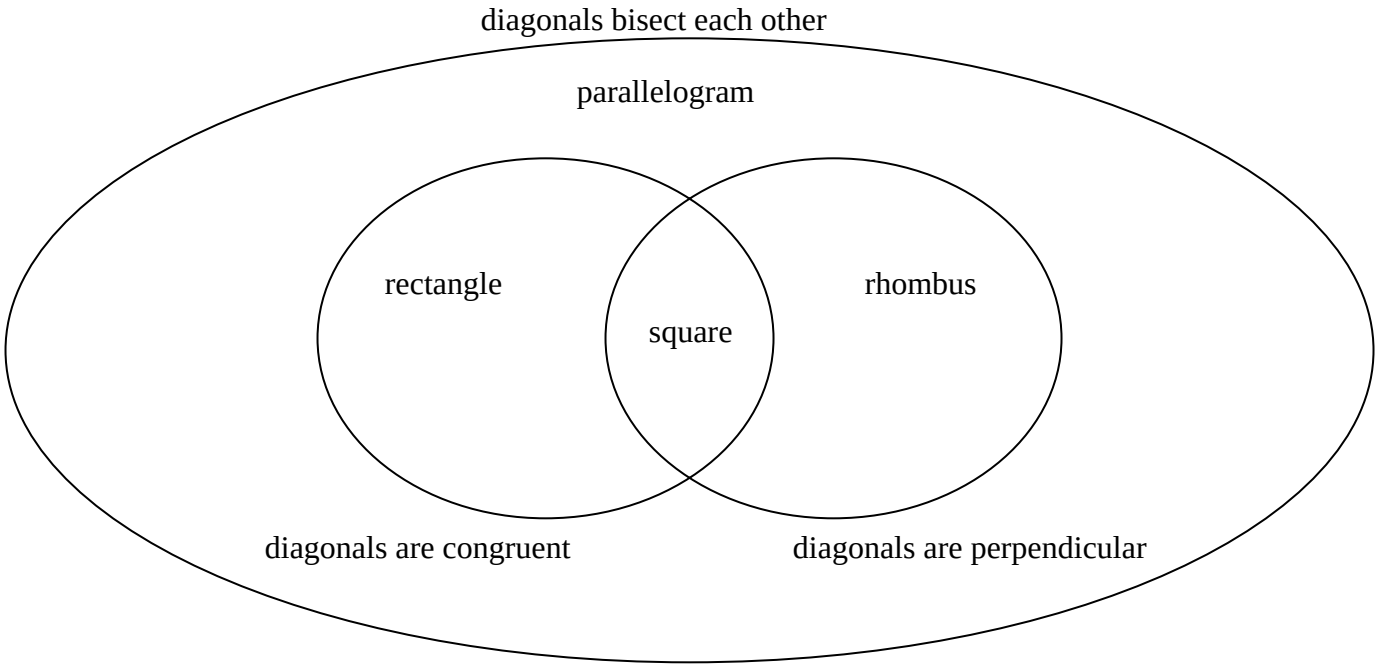
If the diagonals of a quadrilateral __bisect each other _____, then the quadrilateral must be a parallelogram.

If the diagonals of a quadrilateral __bisect each other and are congruent__, then the quadrilateral must be a rectangle.

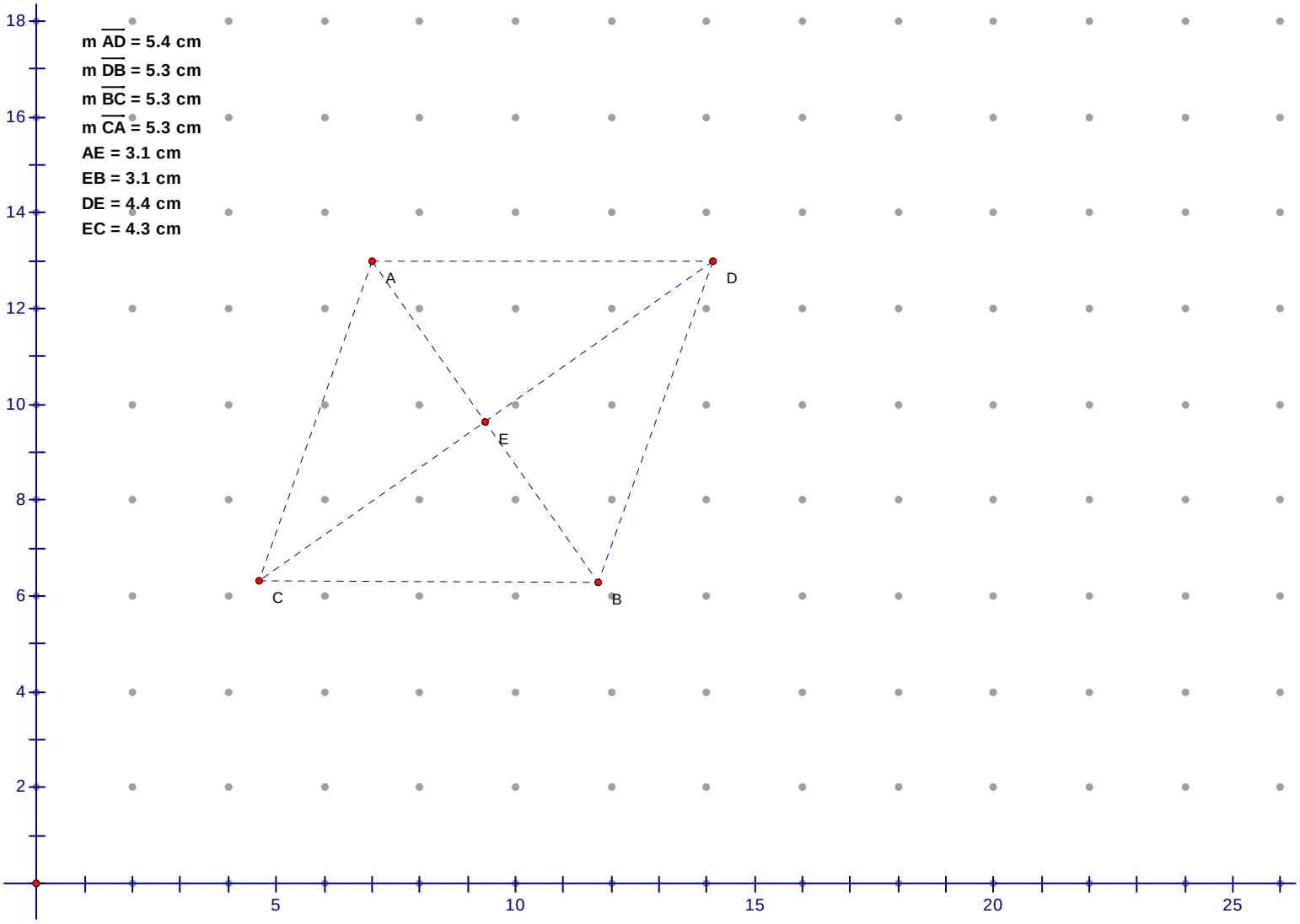
If the diagonals of a quadrilateral __bisect each other and are perpendicular___, then the quadrilateral must be a rhombus.

If the diagonals of a quadrilateral _bisect each other, are congruent and are perpendicular_, then the quadrilateral must be a square.

Bonus: create a Venn diagram showing the relationships between the quadrilaterals based on the characteristics of their diagonals.



TM 21


m $\overline{AD}$ = 5.4 cm
m $\overline{DB}$ = 5.3 cm
m $\overline{BC}$ = 5.3 cm
m $\overline{CA}$ = 5.3 cm
AE = 3.1 cm
EB = 3.1 cm
DE = 4.4 cm
EC = 4.3 cm
A
D
E
C
B
18
16
14
12
10
8
6
4
2
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10
15
20
25

TM-22

PROBLEM-BASED INSTRUCTIONAL TASKS

- Help students develop a deep understanding of important mathematics
- Are accessible yet challenging to all students
- Encourage student engagement and communication
- Can be solved in several ways
- Encourage the use of connected multiple representations
- Encourage appropriate use of intellectual, physical and technological tools

Revised September 2005

TM 23

Summarize

1. What are the big ideas covered?
2. What tools did we use? Which tools worked best for which situations?
3. How can these tasks be modified or extended?
4. Can we find real-life applications?
5. What makes this a PBIT?
6. How do the Van Hiele levels impact how these activities are used?

TM 23

Summarize (Responses)

- What are the big ideas covered?
  - *Properties of shapes – particularly a square*
  - *Coordinate system*
- What tools did we use? Which tools worked best for which situations?
  - *Geoboard*
  - *Graph Paper*
  - *Ruler and Protractor*
  - *Graphing Calculator*
  - *Geometer's Sketchpad*
- How can these tasks be modified or extended?
  - *Meaningful Distributed Practice could be used to preview and/or review these concepts with the students.*
  - *Extension Activity from Addendum Series (TM 15)*
- What are some real-life applications?
  - *Carpenters making sure they have right angles by measuring diagonals*
- What makes this a PBIT?
  - *Use PBIT Component Sheet (TM 21)*
- How do the Van Hiele levels impact how these activities are used?
  - *Variety of approaches meets the needs of all levels of learners.*

Activity 6: Closure

Time: 15 minutes

Overview and Rationale:
This activity ties together the day.

Conducting the Activity

1 Review goals and activities of the day
2 Review homework assignment for next meeting
3 Pass out Evaluation form

Materials
TM 1: Overview
TM 24: Assignment
TM 25: Evaluation

TM 24

ASSIGNMENTS FOR APRIL 11/12:

1. *NCTM Navigating Through Geometry in Grades 6 - 8* – Transformation (pp. 43-44)
   - What are the important ideas in the transformations and symmetry section of the geometry standard?
   - What are some things that might cause problems to students in this area?

2. *NCTM Principles and Standards* – Learning (pp. 20-21) Cognitive research tells us that students learn best when they have an opportunity to build on prior experience and knowledge, develop conceptual understanding during relevant problem solving experiences and use metacognitive reflection.
   - How can these principles enhance student engagement?

3. Slavit, David. "Above and beyond AAA: The Similarity and Congruence of Polygons." on CD from *Navigating through Geometry in Grades 6 – 8* – Article is under section called "More Readings."
   - What impact does this article suggest for curriculum being taught?
   - What impact does this article suggest for instruction?

4. *Driscoll, Mark.* "The Sound of Problem Solving." *Teaching Mathematics through Problem Solving.* (pp. 161 – 175).
   - What can you gain from listening to students when they are working in groups?

5. Brown, Scott. "You Made It through the Test; What about the Aftermath?" *Mathematics Teaching in the Middle School*. (pp. 68 – 73).
   - How does this promote formative use of summative assessment?

6. Do the following activity: Observe a teacher teaching a PBIT
   - List any questions that were asked in class
   - Who asked the questions?
   - Describe the ways the lesson was like a PBIT and the ways it was not
   - How did the teacher assess the students?

TM 25

Every Student Counts

Participant Feedback **Date:**

What is your primary role?

_____AEA Team _____Urban 8 District Team

What were your key learnings from this session?

What questions do you have about the information and content presented and discussed during this session?

What considerations and concerns do you have about your individual use and follow-through of the information presented and discussed this session?

What considerations and concerns do you have about your team use and follow-through of information presented and discussed this session?